

Written Exam for the M.Sc. in Economics, Winter 2011/2012

**Tax Policy**

Final Exam

20 January 2012

(3-hour closed book exam)

### Exercise 1:

Consider a two-period model in which the representative household starts with an endowment  $w$ .

The household enjoys utility over consumption in the two periods,  $c_1$  and  $c_2$ , according to the utility

function  $u(c_1, c_2)$ . The price of consumption is normalized at unity. The interest rate at which the

household can save is  $r > 0$ . Interest income is taxed at an ad valorem rate  $\tau$ .

1.1 Write down the household's budget constraint. Rewrite the budget constraint to see that the capital income tax is equivalent to a tax on second-period consumption.

1.2 Using the tax equivalence result in 1.1, calculate the change in the excess burden following a rise in the tax rate. Calculate the change to a second-order approximation.

1.3 Evaluate the marginal excess burden when  $u(c_1, c_2) = c_1 + \ln c_2$ . Explain how the curvature of the compensated demand function influences the approximated marginal excess burden.

### Exercise 2:

Assume that preferences of the representative household are given by  $U(c_1, \dots, c_N, l)$  where  $c_i$  denotes the consumption level of commodity  $i$  ( $i=1, \dots, N$ ) and  $l$  denotes labor supply. The household budget constraint is  $p_1 c_1 + \dots + p_N c_N = w l$ .  $p_i$  is the consumer price of commodity  $i$  and  $w$  is the wage rate. The consumer price is  $p_i = 1 + t_i$ . That is, the consumer price consists of the producer price (=1) and the tax rate  $t_i$  levied on commodity  $i$ . The household chooses consumption levels and labor supply so as to maximize utility.

The government has to choose the commodity tax rates so as to generate tax revenues at a fixed amount  $R$ .

2.1 Set up the decision problem of the representative household. Characterize the consumption choices and labor supply by means of the first order conditions.

2.2 Formally show the effect of a higher tax rate on household utility once household choices are optimized.

2.3 Now, set up the decision problem of the government and characterize the optimal tax rates by means of the first-order conditions. Explain.

Assume that income effects on consumption demand are negligible and therefore, can be treated as if they were zero. Assume further that the cross-price effects between commodities are zero.

2.4 Characterize the optimal tax rates under these two assumptions. Which feature do the optimal commodity tax rates exhibit?

### Exercise 3:

Consider the Chamley model of capital taxation. The economy consists of a representative household with preferences

$$V_t = \sum_{\tau=0}^{\infty} \delta^{\tau} u(C_{t+\tau}, L_{t+\tau}),$$

where  $C_{t+\tau}$  and  $L_{t+\tau}$  denote consumption and labor supply in period  $t + \tau$ . Markets are competitive and, as usual, the household's optimal choice of consumption between two successive periods follows from the standard Euler condition:

$$\frac{\partial u}{\partial C_t} = \delta(1 + \bar{r}_t) \frac{\partial u}{\partial C_{t+1}},$$

where  $\bar{r}_t$  is the net-of-tax interest rate in period  $t$ . In each period  $t$ , output is produced using the constant return to scale production function  $F_t(K_t, L_t)$  where  $K_t$  denotes the capital stock in period  $t$ . Capital does not depreciate in production. Hence the resource constraint of the economy is

$$F_t(K_t, L_t) + K_t = C_t + G_t + K_{t+1}.$$

$G_t$  is government consumption in period  $t$ . The government can issue public debt and levies a linear tax rate on capital and labor. Thus, the net-of-tax wage rate and interest rate the household receives are  $\bar{r}_t = (1 - \tau^k)r_t$  and  $\bar{w}_t = (1 - \tau^l)w_t$ . The intertemporal budget constraint of the government is

$$b_{t+1} = (1 + \bar{r}_t)b_t + \bar{r}_t K_t + \bar{w}_t L_t - F(K_t, L_t) + G_t.$$

3.1. Write down government tax revenues  $R_t$  in period  $t$ .

3.2. Show that tax revenues can be written as

$$R_t = (r_t - \bar{r}_t)K_t + (w_t - \bar{w}_t)L_t.$$

3.3. Write the Lagrangian of the government's decision problem.

3.4. Given the fiscal instruments at hand, the government effectively chooses the capital stock of the economy. Write down the first-order condition for the capital stock  $K_{t+1}$ .

3.5. Evaluate the first-order condition on a stationary trajectory.

3.6. Determine the optimal difference  $r_t - \bar{r}_t$ . Provide an economic intuition for your finding.

#### Exercise 4:

Consider the Mirrlees model of optimal income taxation, as formulated in Saez (2001), with the following assumptions and notation:

- Assume a quasi-linear utility function  $u(c,l)=c-v(l)$ , where  $c$  is consumption and  $l$  is labor supply.
- Let  $H(z)$  be the cumulative density function of income  $z=wl$  [population is normalized to 1] and  $h(z)$  its density.  $w$  is the wage rate.
- Let  $g(z)$  denote the social marginal value of consumption for taxpayers with income  $z$  expressed in terms of public funds.
- Let  $G(z)$  be the average social marginal value of consumption for taxpayers with an income above  $z$ , i.e.  $G(z)=\int_z^{\infty} g(s)h(s)ds / 1 - H(z)$ .

4.1 Consider a small reform of the tax schedule. That is,  $T'$  increases by  $d\tau$  in a small income band  $(z, z+dz)$ . Derive formally the optimal income tax schedule. Use a perturbation argument and show formally each effect which enters the first-order condition and, thereby, the optimal tax schedule.

4.2 Provide an economic intuition for each term in the first-order condition. Be precise when describing the tax schedule in terms of its marginal tax rates  $T'$ .

4.3 Show formally why an Earned Income Tax Credit (EITC) is not optimal in the Mirrlees model. Explain.